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Sensitivity and Insensitivity of Galaxy Cluster Surveys to New Physics

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Abstract: We study the implications and limitations of galaxy cluster surveys for constraining models of particle physics and gravity beyond the Standard Model. Flux limited cluster counts probe the history of large scale structure formation in the universe, and as such provide useful constraints on cosmological parameters. As a result of uncertainties in some aspects of cluster dynamics, cluster surveys are currently more useful for analyzing physics that would affect the formation of structure than physics that would modify the appearance of clusters. As an example we consider the ΛCDM cosmology and dimming mechanisms, such as photon-axion mixing.

Keywords: Large Scale Structure, Galaxy Cluster Surveys.

Contents

1. Introduction

In this era of precision cosmology, a wide variety of cosmological and astrophysical observations are providing strong constraints on the composition of our universe. Among these are studies of the cosmic microwave background (CMB) [1], large scale structure [2], luminosity-redshift curves of Type Ia supernovae [3], galaxy rotation curves [4], and light element abundances [5]. A relatively consistent picture of the universe has emerged in which the current universe is flat ($\Omega = 1$), contains about 20% of its energy density in nearly pressureless cold dark matter, about 76% in dark energy ($\Omega_\Lambda = .76$), and the remainder in ordinary matter described by the Standard Model of Particle Physics (SM) [1]. (We take Ω_m to be the sum of the cold dark matter and Standard Model matter, including neutrinos, so $\Omega_m = 0.24$ by the above estimates.) The flatness of the universe and the spectrum of initial density perturbations is explained by the paradigm of inflation. On the other hand, dark matter and dark energy provide a challenge for particle physics. The influence of dark matter on galaxy rotation curves, the CMB, and most directly in the observed separation of dark matter and baryonic matter in the “Bullet cluster” [6], provides conclusive evidence that there are

new types of particles which have not been observed in particle physics experiments and are not described by the SM; and it may be argued that the observation of dark energy in the expansion history of the universe hints at new gravitational physics.

The incredible precision of lunar ranging measurements produce some of the strongest constraints on new gravitational physics [7], but only on local phenomena that would be occuring here and now. The overall expansion history of the universe constrains the influence of new physics on the largest of scales, and indeed the luminosity-redshift curves of Type Ia supernovae have provided the most direct evidence for dark energy. The formation of structure in the universe is also highly dependent on gravitational and particle interactions, and since structure has had a relatively long time to form, galaxy and cluster surveys provide another useful probe of the amount and features of dark matter and dark energy, as well as other new physics. The purpose of this paper is to examine the importance of galaxy cluster surveys in testing of new ideas in gravitational and particle physics. (See also [8, 9].) It is certainly not a new idea to use structure formation to constrain cosmological models. Indeed, the Press-Schechter formalism for predicting counts of virialized objects is more than 30 years old [10]. Clusters are the largest virialized objects in the universe, and as such provide a useful probe of structure formation. Collisions of hydrogen atoms in the intracluster gas produce X-rays, and track the gravitational potential well in a cluster [11]. As a result, X-ray surveys have provided reliable and complete surveys of galaxy clusters in various regions of the sky. Serious studies of the properties of X-ray clusters for this purpose began in the 1980's [12, 13, 14, 15]. It was suggested by some groups that cluster surveys were in conflict with the Concordance Model ($\Omega_m = 0.3, \sigma_8 \sim 0.9$) [16, 17, 18, 19]. It now seems that the HIFLUGCS cluster survey is in agreement with the most recent Cosmic Microwave Background data ($\Omega_m = 0.234$ and $\sigma_8 = 0.76$) [1, 20]. Our analysis with the ROSAT 400 Square Degree data set is also in agreement with CMB data if a large scatter is assumed in the relation between cluster mass and temperature. Otherwise, our analysis prefers cosmological parameters closer to the old Concordance Model.

The particle physics community has not yet embraced cluster technology for the purpose of testing physics beyond the standard model. In large part this is because of limited statistics and uncertainties in the theoretical models of structure formation and cluster dynamics. However, while supernovae are sensitive to the geometry (and opacity) of the universe, while structure growth is sensitive to its clustering properties, these are truly complementary approaches, as argued by Wang et. al [21]. However, since cluster surveys can provide constraints on new physics complementary to other cosmological constraints, they deserve to be in the arsenal of the particle physics trade. A purpose of this paper is to review and introduce much of the technology involved to the particle physics community. As an example of the application of cluster surveys to particle physics and its limitations, we study the significance of current and future surveys for constraining dimming mechanisms such as the photon-axion oscillation model of Csáki, Kaloper and Terning [22]. To motivate consideration of dimming mechanisms, we note that while there are numerous models of particle physics beyond the SM which provide dark matter candidates, the nature of the dark energy is more of a mystery.

Constraints on the dark energy equation of state from WMAP and the Supernova Legacy Survey (SNLS [23]) suggest that, assuming a flat universe, $w = -0.97 \pm 0.07$ [1], where $p = w\rho$ is the linearized equation of state relating the pressure of the dark energy fluid p to its energy density ρ. The value $w = -1$ describes the vacuum energy, or cosmological constant. However, naive particle physics estimates of the vacuum energy are dozens of orders of magnitude too large, so it is well motivated to consider alternative models.

If a new pseudoscalar particle existed with a certain range of mass and axion-type coupling to the electromagnetic field, then distant objects would appear dimmer than expected because a fraction of the light emitted by the stars in a galaxy would have been converted to axions while traversing the intergalactic magnetic field [22]. It would be necessary to reevaluate the evidence for acceleration of the universe if the dimming of distant supernovae could be explained without a cosmic acceleration. At the time when the photon-axion oscillation model was proposed, a universe without acceleration could not be ruled out if one allowed for such a dimming mechanism. Since that time, new data has provided stronger constraints on the equation of state parameter of the dark energy, and a model without acceleration is currently disfavored [1]. However, photon-axion oscillations (or any other viable dimming mechanism) could still exist, and would lead to an apparent decrease in the dark energy equation of state parameter, a possibility which remains open [24].

A dimming mechanism would also affect flux limited galaxy cluster surveys. Some distant galaxy clusters which would have otherwise been bright enough to be detected in a flux limited telescope, may become too dim to be detected as a result of photon loss, but it is not a priori obvious what the implications for cosmological analyses would be. Although the appearance of clusters would be affected by such dimming, such effects can be absorbed into the measured evolution of the luminosity-temperature relation. At any rate, clusters provide an independent test of the nature of dark energy which is complementary to supernovae, and thus potentially constraining of models such as dimming mechanisms.

In the following, we will attempt to provide a thorough review of how cosmology relates to theories of structure formation and our observations. Our analysis relies on several assumptions regarding cluster luminosities and their evolution. A better theoretical understanding of the evolution of cluster properties is desirable (however, see Ref. [25]). On the other hand, since the apparent evolution of cluster luminosities has been measured [26, 27], cluster counts provide more direct constraints on new physics that would affect the formation of clusters rather than their appearance. We will use the 400 Square Degree ROSAT survey [28] (hereafter referred to as 400d) as our primary data set. We also use the 400d survey to constrain the standard ΛCDM cosmology, which does not require a theoretical understanding of the mechanisms of luminosity evolution.

In Section 2 we review the statistical models of structure formation based on the Press-Schechter formalism [10]. In Section 3 we analyze the possibility of photon-axion oscillations in light of current galaxy cluster surveys. Interestingly, while supernovae surveys can be dramatically affected by dimming, because the redshift evolution of luminosity and temperature is measured, the studies of cluster count evolution are remarkably insensitive to it (although

the total counts, themselves, are). In Section 4 we present our statistical analysis of cluster constraints for standard cosmology, and thus demonstrate the techniques which are simply applied to other theories of modified dark matter or dark energy. In Section 5 we examine the significance of future cluster surveys for probing new gravitational and particle physics. We conclude in Section 6.

2. Analytical Models of Structure Formation

Although the state of the art in structure formation involves elaborate n-body simulations, much can be understood within simple, analytical models. In this section we summarize the basic theory behind structure formation in the universe and how the theory is compared with galaxy cluster surveys. The review is simplified, and does not contain new results, but we hope it contains enough of the basic ideas so that particle physicists can easily apply the formalism to constrain new physics. There are a number of excellent reviews on structure formation in the literature that are substantially more comprehensive than this one, such as Refs. [29]. Techniques for comparing the models to X-ray cluster data are somewhat scattered in the literature, though recent cluster surveys provide useful background with their catalogues, as in Refs. [17, 28, 30]. Our goal is to simplify the discussion to its bare essentials without forfeiting too much of the underlying physics, making use of the fact that others have performed the complicated simulations necessary to test both the phenomenological models of hierarchical collapse, and hydrodynamic scaling relations between cluster mass and observational quantities like cluster temperature. Simulations suggest that the simplified models of structure formation and X-ray cluster dynamics are accurate enough to constrain new physics by building the new physics on top of these models. While simulations are not in perfect agreement with these models [31], agreement is good enough that these models serve as a useful tool in studying the evolution of structure.

2.1 The Press-Schechter Formalism

The CMB provides strong evidence that the universe was homogenous to a part in 10^5 at the time that atoms formed during recombination. However, as the universe expanded structure formed due to the gravitational collapse of these small fluctuations into progressively larger objects. The precise way in which structure formation occurs is sensitive to the composition of the universe. Smooth, unclustering dark energy, for example, leads to a faster expansion of the universe and hinders formation of structure on large scales. Since the evolution of structure depends on the composition of the universe, comparison of models to observations provides an important probe of cosmology. The Press-Schechter (PS) formalism [10] provides a simple model for translating cosmology into number counts for structures on arbitrary length scales, as a function of mass and redshift. Here we summarize only the main results of this formalism, but there are many lengthier discussions in the literature justifying this approach and deriving the relevant results quoted below (*e.g.* Refs. [29]).

The spherical collapse model of Press and Schechter imagines that initially overdense regions of the universe collapse with spherical symmetry, in an otherwise homogeneous universe. As the universe expands, only objects with a density above a critical value will have collapsed and virialized by any given time. For the remainder of this discussion we assume a flat universe with $\Omega_m + \Omega_\Lambda = 1$. In terms of the cosmological parameter $\Omega_f(z)$, where

$$\Omega_f(z) = (\Omega_m\,(1+z)^3)/(\Omega_m(1+z)^3 + (1-\Omega_m)), \tag{2.1}$$

the critical overdensity at the time of virialization in the linearized spherical collapse model is given by [32]:

$$\delta^v_{sc} \simeq \frac{3\,(12\pi)^{2/3}}{20}\,(1 + 0.0123\log_{10}\,\Omega_f)\,. \tag{2.2}$$

In the Einstein-de Sitter universe with $\Omega_m = 1$, $\delta^v_{sc} \approx 1.69$. In order to compare with the density field observed today, we need to account for the evolution of the universe. Since virialized objects are nonlinear fluctuations of the background density field, it would seem difficult *a priori* to describe analytically the evolution of the statistical distibution of density perturbations. It was the observation of Press and Schechter that since the original spectrum of density perturbations was approximately Gaussian, and because the precise nonlinear evolution of those density perturbations is unlikely to significantly modify the *mass* contained in collapsed objects, a linearized approach can be justified for modeling the distribution of massive collapsed objects.

In the linear model, density perturbations grow proportional to the growth factor $D(z)$, and thus the linearized overdensity of an object that virialized at a redshift z has grown by

$$\delta_c\,(z) = \delta^v_{sc}\,D(0)D(z)^{-1}, \tag{2.3}$$

where the linear growth factor is defined as [33, 34],

$$D(z) = 2.5\,\Omega_m H_0^2 H(z) \int_z^\infty \frac{(1+z')}{H(z')^3} dz'\ , \tag{2.4}$$

and $H(z)$ is the Hubble parameter at redshift z,

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \tag{2.5}$$

Below we will use $H_0 = 100h$ km/s/Mpc with $h = 0.73$ [1].

If one assumes a Gaussian distribution, the probability of a given collapsed object of mass M having an overdensity in the linearized model larger than $\delta_c\,(z)$ today is

$$p(\delta_c\,(z)\,, M) = \frac{1}{\sqrt{2\pi}\sigma}\int_{\delta_c(z)}^\infty \exp(-\delta^2/2\sigma^2) d\delta. \tag{2.6}$$

By differentiating $p(\delta_c, M)$ with respect to M and dividing by the volume $(M/\bar{\rho})$ one gets the number density of objects with mass between M and $M + dM$. The present day mean matter density of the universe is $\bar{\rho} = 2.775 \times 10^{11}\,\Omega_m\,h^2\,M_\odot\,\mathrm{Mpc}^{-3}$[35]. In this simplified picture of

structure formation, small objects become bound first, followed by larger structures. Galaxy clusters are the largest virialized objects in the universe in the current epoch, which makes them especially suitable for study by this approach.

The variance in the distribution of density fluctuations in the universe, $\sigma(M)^2 = \langle(\delta M/M)^2\rangle$, is typically normalized to spheres of radius $8\,h^{-1}$Mpc:

$$\sigma\left(M\right)^2 = \sigma_8^2\,\frac{\int_0^\infty k^{2+n}\,T(k)^2\,\left|W\left(kR(M)\right)\right|^2 dk}{\int_0^\infty k^{2+n}\,T(k)^2\,\left|W\left(k\,8h^{-1}\,\mathrm{Mpc}\right)\right|^2 dk} \tag{2.7}$$

where σ_8 is fit to cosmological data. The function $W(x)$ is a top-hat filter, $W(x) = 2\left(\sin x - x\cos x\right)/x^3$, so that $\sigma(M)$ is the variance of the mass distribution in spherical volumes of radius $R\left(M\right) = \left[3M/\left(4\pi\overline{\rho}_0\right)\right]^{1/3}$. The initial power spectrum is usually assumed to take the scale-free Harrison-Zel'dovich form $P(k) \propto k^n$, with $n = 1$. As the universe expanded density perturbations grew at different rates through the radiation dominated era to the present era. Likewise different size fluctuations crossed the particle horizon at different times. The power spectrum therefore varies from the Harrison-Zel'dovich spectrum in a wavelength dependent way. Assuming linearity, *i.e.* an absence of mixing of modes with different wavenumbers, the power spectrum evolved from early times may be written as $P(k) = T(k)^2 k$,[1] where the transfer function, $T(k)$, takes the primordial power spectrum to the present day. As $k \to 0$, $T(k) \to 1$ [37] because large enough wavelength fluctuations have not crossed the particle horizon and therefore keep their primordial spectrum. This also means that the initial time can be taken to be any time before which fluctuations of interest would have crossed the horizon. The form of the transfer function is found by analyzing numerically physical processes that would modify the power spectrum over time. For cold dark matter, Bardeen, *et al.* [38] found,

$$\begin{aligned} T\left(k\right) &= \ln\left(1 + 2.34q\right)/\left(2.34q\right) \\ &\quad\times \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4} \end{aligned} \tag{2.8}$$

where $q(k) = k/\left(\Omega_m h^2\,\mathrm{Mpc}^{-1}\right)$ in the absence of baryonic matter. To account for baryon density oscillations a "shape parameter" Γ is introduced, simply replacing $q(k)$ by [39],

$$q(k) = \frac{k}{\Gamma\,h\,\mathrm{Mpc}^{-1}}, \tag{2.9}$$

where [40],

$$\Gamma = \Omega_m h\,\exp\left[-\Omega_b(1 + \sqrt{2h}/\Omega_m)\right], \tag{2.10}$$

and Ω_b is the ratio of the baryon density to critical density.

While the Press-Schechter formalism is remarkably successful in its comparison to numerical simulations ([10, 41, 42]), it has proven to be most powerful as a basis for a phenomenological approach to modeling galaxy cluster counts. One extension to the formalism takes

[1] We absorb the superhorizon evolution into $P(k)$; see Ref. [36] for a discussion.

into account non-sphericity of collapsing objects. Sheth, Mo and Tormen (SMT) developed a modified PS procedure [41] which, allowing for ellipsoidal collapse, introduces new model parameters which are fit to N-body simulations. In the SMT model, the mass function is given by

$$\frac{dN}{dM} = \sqrt{\frac{2a}{\pi}}\, c\, \frac{\bar{\rho}}{M}\frac{d\nu}{dM}\left(1+\frac{1}{(a\nu^2)^p}\right)\exp\left(-\frac{a\nu^2}{2}\right), \tag{2.11}$$

where $\nu = \delta_c(z)/\sigma(M)$, and the best fit for the parameters a, c and p assuming a standard ΛCDM cosmology are $a = 0.707$, $c = 0.3222$, $p = 0.3$ [41]. (By comparison, in the PS model, $a = 1$, $c = 0.5$ and $p = 0$.)

2.2 Relating Measured Flux to Cluster Luminosity and Mass

The Press-Schechter formalism and its extensions reviewed above predict the statistical distribution of massive collapsed objects in the universe as a function of their masses and redshifts. On the other hand, telescopes do not directly measure cluster masses, but rather the flux and perhaps the spectrum of light emitted by those clusters in some frequency band as observed on or near Earth. In order to relate the mass function (2.11) to observational quantities, it is necessary to understand the relationships between the mass of a cluster and observational data. In this section we describe how properties of X-ray clusters are related to one another, and how those properties are then compared with observations.

On average, hydrodynamical models which yield simple scaling relations between cluster mass and cluster temperature (the $M - T$ relation) have been proven reasonably successful in comparison with numerical simulations [35, 43]. On the other hand, the relation between the temperature and X-ray luminosity (the $L - T$ relation) of clusters is sensitive to more complicated physics such as cooling mechanisms and the density profile of the intracluster gas, and is fit by cluster data. Furthermore, it is now commonly accepted that the $L - T$ relation has evolved as the composition of radiating cluster gases has evolved [26].

There are at least two sensible notions of cluster temperature, so it is important to be precise in terminology. From here on when we refer to a cluster's temperature, T, we will mean the temperature of the baryonic gas in the cluster, as is directly measured from the spectrum of light emitted by the gas in the cluster. We model the cluster gas as isothermal, which may not be that good an approximation for actual clusters [44], although predictions for number counts are not that sensitive to this assumption [35]. Another notion of cluster temperature is determined by the velocity dispersion of the dark matter particles, $\sigma^2 = \langle v^2 \rangle$, where the velocity v is measured in the rest frame of the cluster and the brackets denote the statistical average over dark matter particles. If typical dark matter particles have a mass m_D, then the quantity $T_D \equiv m_D\sigma^2/k_B$ is a measure of the temperature of the dark matter in the cluster, where k_B is Boltzmann's constant. Generally T_D is not directly related to T, as the dark matter is not expected to be in equilibrium with the baryonic matter. However, it is often assumed that these temperatures are similar, or at least proportional to one another, after which a scaling relation between cluster temperature and cluster mass follows.

For an isothermal spherical cluster of dark matter, the density ρ and velocity dispersion σ scale with distance from the cluster center r as [45]:

$$\rho(r) = \frac{\sigma^2}{2\pi G_N r^2}, \tag{2.12}$$

with Newton's constant G_N. As mentioned earlier, the assumption of isothermality may not accurately describe the density profile of the halo, which is a subject of intense study. A phenomenological density profile which fits better simulations is given by the model of Navarro, Frenk and White (NFW) [46], in which the density profile takes the form,

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}, \tag{2.13}$$

where ρ_s and r_s are model parameters. In the current analysis we assume the isothermal profile, Eq. (2.12), for easier comparison to analytic approximations of scaling relations in the literature.

By considering the evolution of spherical density perturbations, one can estimate the density of objects which had just virialized at redshift z. The density of virialized objects may be written in terms of $\Delta(z)$, the ratio of the cluster density to the critical density, $\rho_{crit} = 3H^2/8\pi G$. Assuming unclustering dark energy, and ordinary CDM, a useful analytic approximation to $\Delta(z)$ was given in Ref. [47] for flat ΛCDM cosmologies:

$$\Delta(z) = 18\pi^2 + 82(\Omega_f(z)-1) - 39(\Omega_f(z)-1)^2, \tag{2.14}$$

where $\Omega_f(z)$ is given by Eq. (2.1). A scaling relation between the velocity dispersion and the cluster mass is obtained by approximating the mass of a spherical cluster which virialized at redshift z to the mass obtained by integrating Eq. (2.12) to a radius such that the mean density is given by $\rho_{crit}\,\Delta(z)$, with the result [47],

$$\sigma^2 \sim M^{2/3} H(z)^{2/3} \Delta(z)^{1/3}. \tag{2.15}$$

Here, $H(z)$ is the redshift-dependent Hubble parameter, Eq. (2.5). Assuming the baryonic gas in a cluster has temperature proportional to the dark matter velocity dispersion σ^2, it follows that the cluster temperature, T, scales with cluster mass, M, and redshift, z, as in Eq. (2.15):

$$T \sim M^{2/3} H(z)^{2/3} \Delta(z)^{1/3}. \tag{2.16}$$

In practice, simulations are used to determine the constant of proportionality T_{15} defined through [17],

$$T = T_{15}\left(\frac{h}{0.73}\right)^{2/3}\left(\frac{\Omega_m \Delta(z,\Omega_m)}{178\,\Omega_f(z)}\right)^{1/3}\left(\frac{M}{10^{15} M_\odot}\right)^{2/3}(1+z)\ , \tag{2.17}$$

where $\Omega_f(z)$ is given by Eq. (2.1), and $M_\odot$ is the solar mass. The normalization factor 178 is approximately the overdensity of a just-virialized object (*c.f.* Eq. (2.2) in the linear model).[2]

Different simulations determine a variety of values for T_{15}, which leads to some ambiguity as to the most accurate normalization for the $M-T$ relation. Typical values are $T_{15} \approx 4.8$ keV and $T_{15} \approx 5.8$ keV [47]. We do fits for various values of T_{15} to gauge the errors associated with the uncertainty in the $M-T$ relation.

With a relation between cluster temperature and cluster mass in hand, the SMT mass function, Eq. (2.11), can be used to determine how many clusters of a given temperature are expected per unit volume of the sky as a function of redshift. However, telescopes often have poor spectroscopic resolution, so that in many X-ray cluster surveys it is difficult to accurately determine cluster temperatures. Furthermore, telescopes are unable to observe arbitrarily dim objects, *i.e.* they are flux limited. Hence, in order to use the Press-Schechter formalism to predict observed number counts of galaxy clusters it is still necessary to relate the cluster temperature to observed flux. Such a relationship comes in the form of the elusive $L-T$ relation [14, 26]. There are a number of complications in predicting, and in making practical use of, the $L-T$ relations which appear in the literature: (*i*) Surveys often quote fluxes in some frequency band, not the bolometric (*i.e.* total) flux. X-ray telescopes are sensitive to light with frequencies of fractions of a keV to tens of keV, though not always in precisely the same frequency band. (*ii*) The measured frequency band is specified in the telescope's reference frame, so redshifting of the sources affects the fraction of the total luminosity observed in the specified frequency band. (*iii*) There is some scatter in the $L-T$ data (from which $L-T$ relations are fitted), which is due in part to a complicated cooling process that takes place in many clusters in the central region of the cluster gas [14]. As a result, when possible, some surveys remove the central cooling regions when inferring X-ray luminosities, and some do not. (*iv*) In addition, there is relatively strong evidence that the $L-T$ relation has evolved over time [26] due to changing cluster environments. (*v*) Furthermore, when inferring luminosities of distant objects from measured fluxes a particular cosmology must be assumed, and the assumed cosmology may differ from one quoted evolving $L-T$ relation to another.

In this paper we focus on the recent 400d ROSAT survey [28], so we will make use of published $L-T$ data most easily compared to the cluster luminosities as presented by the 400d survey. In particular, the 400d survey quotes X-ray fluxes in the 0.5-2 keV band including the central cooling regions. We begin with the $L-T$ relation determined by Markevitch [14] from 35 local ($z < 0.1$) clusters. The fitted power law $L-T$ relation takes the form

$$L^{local}_{0.1-2.4} = A_6 \left(\frac{T}{6\,\mathrm{keV}} \right)^B , \tag{2.18}$$

[2]Refs. [17, 48] define $\Delta(z)$ as the contrast density with respect to the *background* density at redshift z. As in Ref. [47], we are defining the contrast density with respect to the *critical* density, $\rho_{crit} = 3H^2/8\pi G$. This is the origin of the different scaling relations as written in Ref. [47] and in Refs. [17, 48]. Physically, they are equivalent, assuming an isothermal density profile.

with $A_6 = (1.71 \pm 0.21)\, 10^{44} h^{-2}$ erg s^{-1}, and $B = 2.02 \pm 0.40$, where cooling flows were not removed when inferring either luminosities or temperatures [14].

To study the redshift dependence of the $L-T$ relation, Vikhlinin, *et al.* [26] measured the X-ray temperature and fluxes of 22 clusters at redshifts $0.4 < z < 1.3$ and with temperatures between 2 and 14 keV. The luminosity L inferred from the flux F depends on the assumed cosmology via,

$$L = 4\pi\, d_L(\Omega_m, z)^2\, F\, K(z), \tag{2.19}$$

so the observed redshift dependence of the $L - T$ relation depends on the cosmology. The K-correction $K(z)$ will be discussed below. The luminosity distance, d_L, is given by,

$$d_L(\Omega_m, z) = (1 + z) \int_0^z \frac{c\, dz'}{H(z')}. \tag{2.20}$$

The integral is the comoving distance between the source and the telescope. The extra factor of $(1+z)^2$ in d_L^2 accounts for the decreased energy per photon from redshifting of the source, and the decrease in frequency between photon arrival times, as the universe has expanded. To correctly interpret the luminosity evolution in different cosmologies, Eq. (2.18) should be modified, assuming power law evolution, with the reference cosmology factored out:

$$L_{0.1-2.4} = A_6 \left(\frac{T}{6\,\mathrm{keV}}\right)^B \frac{d_L(\Omega_m, z)^2}{d_L(1, z)^2}\, (1 + z)^\alpha. \tag{2.21}$$

Vikhlinin, *et al.* found that assuming a $\Omega_m = 1$, $\Omega_\Lambda = 0$ reference cosmology, $\alpha = 0.6 \pm 0.3$. It is important to stress that a nonvanshing power α does not in itself imply an evolution of cluster properties, because α is cosmology dependent. However, assuming a more realistic $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ reference cosmology leads to a still larger power, $\alpha_{\Omega_m=0.3} = 1.5 \pm 0.3$ [26]. Hence, it seems difficult to argue that the inferred luminosity evolution is due to a mistaken assumption about the cosmological expansion rate. We also note that other surveys find similar results. For example, the XMM Omega project determined $\alpha = 0.65 \pm 0.21$ [17].

We need to be able to convert between frequency bands both as a result of the redshifting of the spectrum, and in order to compare measurements of surveys in different frequency bands. From Eq. (2.21), we can infer a similar relation for the luminosity in the 0.5-2 keV band (as appropriate for the 400d survey) in the cluster rest frame if we know the X-ray spectrum. The difficulty is that spikes in the spectrum from atomic excitations contribute significantly to the flux, so some understanding of the components of the cluster gas is necessary to accurately convert luminosity in one frequency band to luminosity in another frequency band. A popular and accurate model is the optically thin plasma model of Mewe, Kaastra, Liedahl, and collaborators [49], the so-called Mekal model. The Chandra Interactive Analysis of Observations (CIAO) software package [50] contains code for the purpose of converting spectra between bands and between reference frames, and includes packaged spectral models. The REFLEX cluster survey has also tabulated conversion factors between luminosities in various frequency bands [30] as a function of temperature, for easy comparison of cluster data to structure formation models without necessitating installation of the CIAO software.

We assume that the luminosity evolution parametrized by the $(1+z)^{\alpha}$ dependence in Eq. (2.21) is uniform across the spectrum, so that the same power α will describe evolution of the 0.1–2.4 keV $L-T$ relation as in the 0.5-2 keV $L-T$ relation. As a test of this assumption we studied the $z > 0.4$ cluster data by Vikhlinin *et al.* [26], which includes measurements of flux in the 0.5-2 keV band and bolometric flux. We checked that Vikhlinin *et al.*'s fit of $\alpha \approx 0.6$ is valid both with their measured bolometric fluxes and fluxes in the 0.5-2 keV band. For our fits we use,

$$\begin{aligned} \alpha &= 0.6 \quad \text{(with an } \Omega_m = 1 \text{ reference cosmology)} \\ A_6 &= 1.06 \quad \text{(for the } 0.5-2 \text{ keV band)} \\ B &= 2.02 \end{aligned} \tag{2.22}$$

as the parameters in Eq. (2.21).

The final factor required to compare intrinsic luminosity to measured flux is the K-correction. The K-correction converts from the luminosity in a specified frequency band in the rest frame to luminosity in the same frequency band in the lab frame, as per Eq. (2.19). The K-correction for a source at redshift z in the frequency band (f_1, f_2) is given by,

$$K(T,z) = \frac{\int_{f_1}^{f_2} df\, P_T(f)}{\int_{f_1(1+z)}^{f_2(1+z)} df\, P_T(f)}, \tag{2.23}$$

where $P_T(f)$ is the rest frame spectral distribution for an X-ray cluster with temperature T, as a function of frequency f. For example, the measured flux F in the 0.5-2 keV band from a cluster at redshift z with rest frame luminosity $L_{0.5-2}$ and temperature T is given by,

$$F = \frac{L_{0.5-2}}{4\pi d_L(\Omega_m, z)^2} \frac{\int_{0.5(1+z)}^{2(1+z)} df\, P_T(f)}{\int_{0.5}^{2} df\, P_T(f)}. \tag{2.24}$$

The K-corrections are not strongly temperature dependent except at the low end of typical cluster X-ray temperatures, and a simple power law spectrum,

$$P_T(f) \sim f^{-n}, \tag{2.25}$$

with index $n = 0.5$, is found to give a reasonable fit in the relevant frequency bands [51]. A comparison of the K-corrections from the simple power spectrum and from more precise plasma spectra can be found in Ref. [51], or from the documentation for the Sherpa module of the CIAO software [52]. Since one of our goals is simplicity in comparison of models of new physics to cluster data we will assume the simple power spectrum in our fits. One should keep in mind, however, that without much more difficulty more accurate K-corrections can be obtained using available software.

Finally, in order to predict the number of observed clusters it is necessary to know the probability of the given telescope detecting a cluster with a given flux. The selection function

measures this probability, and depends on the particular survey. The selection function is often presented as an effective sky coverage area as a function of flux, but can easily be converted to a detection probability. The ROSAT 400d survey contained a geometric survey area of $A_{geo} = 446.3$ deg^2 [28]. The selection probability is obtained from their tabulated effective sky coverage, $A_{eff}(f)$ as a function of flux f, via,

$$P_{sel}(f) = \frac{A_{eff}(f)}{A_{geo}}. \tag{2.26}$$

We are now prepared to calculate the number of clusters we expect to see in an area of the sky per unit redshift, for a given telescope flux limit. The mass function Eq. (2.11) gives the distribution of clusters as a function of mass and redshift. We convert mass to temperature using the $M - T$ relation, Eq. (2.17); and then temperature to measured flux in the appropriate frequency band using the $L - T$ relation, Eq. (2.18), K-corrected as in Eq. (2.23), with parameters specified by Eq. (2.22).

3. Dimming Mechanisms and Cluster Counts

3.1 CKT photon loss mechanism

As an example of a dimming mechanism we will study the possibility of photon loss due to photon-axion oscillations (the CKT model [22]) as hypothesized by Csáki, Kaloper and Terning. We first note that both cluster counts and Type Ia supernova surveys extend to comparable redshifts $z \gtrsim 1$. Hence, if the parameters of the CKT model are chosen so as to affect the interpretation of the supernova data, as in [22], then the same dimming mechanism will have an affect on flux limited cluster surveys.

The CKT model assumes an axion-like interaction between a pseudoscalar field $\phi(x)$ and the electromagnetic field of the form,

$$\mathcal{L}_{int} = \frac{1}{M_L\, c^2}\, \phi\, \mathbf{E} \cdot \mathbf{B}, \tag{3.1}$$

where the dimensionful scale M_L governs the strength of the interaction.

There is significant evidence for an intergalactic magnetic field (IGMF), although little is known about its uniformity in magnitude and direction [53]. The typical magnitude of the IGMF is estimated to be 10^{-9} Gauss, and it is typically assumed that the field is coherent to about $L_{dom} \sim 1$ Mpc. In a background magnetic field the photon-axion interaction, Eq. (3.1), gives rise to a mixing between the axion ϕ and the electric field. This mixing causes oscillations just as mixing between neutrino flavors leads to neutrino oscillations. In the case of photon-axion oscillations there is one polarization of the axion and two of the photon, so after traversing enough regions of randomly oriented magnetic fields a beam of photons will become distributed equally among the three polarizations. Asymptotically, the intensity of light received from a distant astrophysical object will be decreased by a factor of 1/3 (in the limit of infinite horizon size). As discussed in [22], this effect is approximately

described by the following expression for the probability of a photon to remain a photon after traversing a comoving distance $r(z)$:

$$P_{\gamma\to\gamma}(z) \simeq \frac{2}{3} + \frac{1}{3}\, e^{-r(z)/L_{dec}}, \tag{3.2}$$

where,

$$L_{dec} = \frac{8}{3} \frac{\hbar^2\, c^6\, M_L^2}{L_{dom}\, |\mathbf{B}|^2}. \tag{3.3}$$

The existence of a dimming mechanism like photon-axion oscillations would modify the interpretation of cluster counts. In particular, there would be a reduction in the number of distant visible clusters in a flux limited observation. Dimming can also mimic luminosity evolution, although photon-axion oscillations cannot explain the observed luminosity evolution. The fact that $\alpha > 0$ in Eq. (2.21) indicates that distant clusters appear more luminous than nearby clusters, while photon-axion oscillations would have led to the opposite conclusion. If we had a theoretically predicted $L - T$ relation, cluster dimming would be accounted for by including a factor of $P_{\gamma\to\gamma}$ in the $L - T$ relation, which would become:

$$L_{bol} = A_6 \left(\frac{T}{6\,\mathrm{keV}}\right)^B \frac{d_L(\Omega_m, z)^2}{d_L(1, z)^2}\,(1+z)^\alpha P_{\gamma\to\gamma}. \tag{3.4}$$

Similarly, the measured flux from a cluster in the 0.5-2 keV band, which was given by Eq. (2.24), would become,

$$F = \frac{L_{0.5-2}\, P_{\gamma\to\gamma}(z)}{4\pi d_L(\Omega_m, z)^2}\, \frac{\int_{0.5(1+z)}^{2(1+z)} df\, P_T(f)}{\int_{0.5}^{2} df\, P_T(f)}. \tag{3.5}$$

However, since dimming mechanisms would mimic luminosity evolution, it is redundant to include the factor $P_{\gamma\to\gamma}$ in Eq. (3.4) if the evolution specified by the parameter α is fit to observations. It is a remarkable fact that, although dimming does affect the appearance of clusters in these surveys, all these effects are absorbed into the z-dependence of the $L - T$ relation. Thus, having measured this evolution, these surveys should determine the nature of dark energy independently of dimming. On the other hand, physics which would affect the formation of structure rather than the appearance of clusters, can be constrained with cluster counts without a theoretical understanding of the evolving $L - T$ relation.

Let us note here two important upshots of this fact: first, given a measured $L - T$ evolution, implications of cluster counts are independent of dimming while supernovae clearly are not. Thus, as data sets expand, comparing these two will constrain any anomalous dimming of supernovae. Secondly, distant clusters tend to be brighter than they would have been in the absense of evolution. With a larger statistical sample of x-ray clusters and a better theoretical understanding of $L - T$ evolution, this alone may be the strongest constraint on dimming mechanisms.

4. Results

In this section we compare number counts found using the above model to the 400d ROSAT survey [28]. The 400d survey identified 242 optically verified X-Ray sources in the main survey. The search was done with a flux limit of 1.4×10^{-13} erg s^{-1} cm^{-2} and with a geometric sky coverage of 446.3 square degrees. In order to compare our theoretical number counts to ROSAT's data, we integrated redshift over a bin size of $\Delta z = 0.1$. Since the 400d survey reported the error bars in their flux measurements, we estimated the error bars on galaxy cluster number counts by counting how many objects in a given redshift bin would lie below the flux limit when the measured flux is shifted downward by one standard deviation. In the cases where an X-ray source lied on the boundary of a redshift bin, it was counted in both of the adjacent redshift bins. To account for the scatter in the $M - T$ and $L - T$ relations, we included a log normal distribution in the effective $L - M$ relation ($L(M, z)$):

$$P_L(\ln L', z) = \frac{1}{\sqrt{2\pi\sigma^2_{\ln L}}} \exp\left[\frac{-\left(\ln(L') - \ln(L(M,z))\right)^2}{2\sigma^2_{\ln L}}\right], \tag{4.1}$$

The effective selection probability $\widetilde{P}_{sel}(M, z)$ of objects of mass M at redshift z is a convolution of the survey selection function $P_{sel}(f)$ with the distribution in luminosities inferred from the $L - M$ relation $P_L(L, z)$:

$$\widetilde{P}_{sel}(M, z) = \int_{\ln L_x(z)}^{\infty} P_L(L', z) P_{sel}\left(\frac{e^{\ln L'}}{4\pi d_L(z)^2}\right) d\ln L', \tag{4.2}$$

where

$$L_x(z) = 4\pi d_L(z)^2 f_x \tag{4.3}$$

is the lower limit on the luminosity at redshift z, corresponding to the flux limit f_x of the survey, and the argument of P_{sel} is the flux expressed in terms of the luminosity and luminosity distance. We compared the best fit values of various observables as the assumed scatter, $\sigma_{\ln L}$, varied from 0.3 to 0.7 [35, 54]. The results are described below, and can also be seen in Fig. 1 and Fig. 3.

The number of observed virialized objects in a redshift bin $\Delta z = 0.1$ is then given by integrating the mass function over objects, weighted by $\widetilde{P}_{sel}(M, z)$:

$$N\left(> f_x, z, \Delta z\right) = A_{geo} \int_{z-\Delta z}^{z+\Delta z} \int_0^{+\infty} dz\, dM\, r(z)^2 \frac{dr}{dz} \frac{dN}{dM} \widetilde{P}_{sel}(M, z), \tag{4.4}$$

where A_{geo} is the geometric sky coverage in steradians and f_x is the flux limit of the survey, which feeds into $\widetilde{P}_{sel}(M, z)$ as described above.

The comoving volume element per steradian is,

$$dV(z) = r(z)^2 \left(\frac{dr(z)}{dz}\right) dz, \tag{4.5}$$

with

$$r(z) = c \int_0^z \frac{dz'}{H(z')}, \tag{4.6}$$

and the Hubble parameter $H(z)$ is given by Eq. (2.5).

It is important to ensure that the lightest mass virialized object included in N given the assumed $M - T$ and $L - T$ relations is cluster sized and not smaller. Otherwise N contains smaller objects which are not included in the survey. This constrains the smallest z for which this formalism is valid. In our fits we only include clusters with redshift $z \geq 2$. The lightest mass virialized object that could have been observed by the 400d survey, with X-ray flux limit $1.4{\times}10^{-13}$ erg/s/cm^2, assuming $\Omega_m = 0.3$ and $h = 0.73$, is around $M(0.2) = 1.6 \times 10^{14} M_\odot$, which is cluster size.

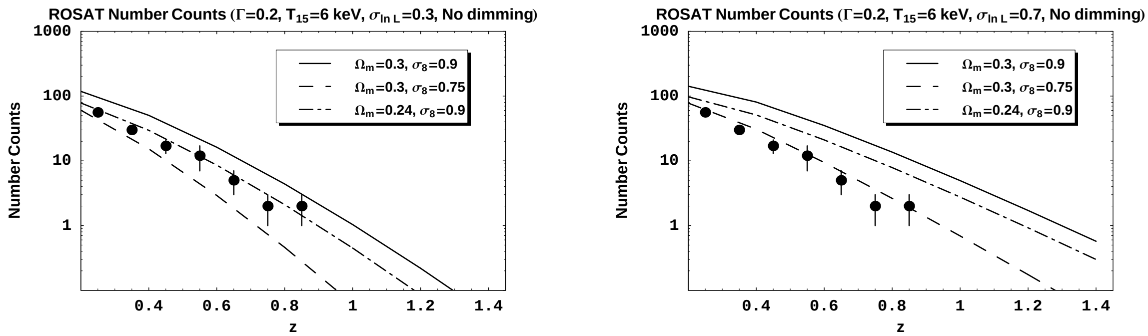
4.1 Systematic Errors

The only errors included in our fits are an estimate of the uncertainty in low flux cluster counts. There are in addition a number of theoretical uncertainties, some of which we studied by repeating our fits with different parameter choices. Larger normalizations of T_{15} in Eq. (2.17) would lead to a prediction of brighter clusters, and hence larger cluster counts. For a fixed data set, larger T_{15} would then translate into a measurement of less structure, corresponding for example to smaller Ω_m and/or σ_8. Similarly, a larger normalization for A_6 in Eq. (2.18) would imply brighter clusters, with similar consequences to increasing T_{15}. To examine the uncertainty in predictions for cosmological and dimming parameters, we repeated our fits for typical determinations of T_{15} from simulations, differing by as much as 20%. The uncertainty in A_6 is effectively equivalent to an additional uncertainty in T_{15} of around 5%. The power laws used in the $L-T$ and $M-T$ relation also have associated errors, and it would be useful to perform a more complete error analysis. Another source of error is our assumed power law spectrum used to calculate K-corrections, which is a worse approximation for low-temperature clusters than high-temperature clusters, but we expect that this is not a significant source of error. We have also checked that alternative redshift binning of the data does not significantly change our results. To examine the effect of scatter on our analysis, we reduced our assumed scatter from $\sigma_{\ln L} = 0.7$ to 0.3 and found that the predicted number counts were reduced by nearly a factor of two, as can be seen below. This demonstrates the importance of correctly accounting for such statistical effects.

4.2 Flux Limited Cluster Counts for Standard Cosmology

Figure 1 shows our computed number counts and the 400d survey's observed number counts versus redshift. Three curves were drawn for different values of Ω_m σ_8, with $\Gamma = 0.2$ and $\mathrm{T}_{15} = 6$ keV.

Larger normalizations for the $L - T$ relation (T_{15}) lead to smaller predicted values of Ω_m and σ_8. Reducing the assumed level of scatter in the effective $L - M$ relation leads to a decrease in the predicted number of dim objects observed. For a given set of observations, reducing the assumed scatter would then lead to a larger inferred amount of structure, *i.e.*



(a) (b)

Figure 1: Number Counts versus redshift for different matter densities (Ω_m) and matter density fluctuation amplitudes (σ_8), without photon-axion oscillations, with different levels of scatter in the L-M relation. The theoretical predictions correspond to $\Gamma = 0.2$ and $\mathrm{T}_{15} = 6$ keV. (a) $\sigma_{\ln L} = 0.3$. (b) $\sigma_{\ln L} = 0.7$.

larger σ_8. For $T_{15} = 6$ keV and $\Gamma = 0.2$, the best fit shifts from $\Omega_m = 0.209$ and $\sigma_8 = 0.923$ with $\sigma_{\ln L} = 0.3$ to $\Omega_m = 0.286$ and $\sigma_8 = 0.731$ with $\sigma_{\ln L} = 0.7$.

Figure 2 shows our χ^2 analysis for different values of T_{15}, Γ and $\sigma_{\ln L}$. We only include our estimated uncertainties in the 400d survey number counts in the statistics. Notice that for values of Γ between 0.1 and 0.2 and T_{15} between 5 and 6 keV, there is tension between our result and the best fit WMAP 3-year measurement. We are consistent with WMAP bounds if we assume large T_{15}, small Γ and large $\sigma_{\ln L}$. Our results are similar to earlier studies [16, 17, 18], although Reiprich [20] has found that the HIFLUGCS cluster survey is in still better agreement with the WMAP 3-year [1] and COBE 4-year data [55]. Flux limited cluster counts can also be used to constrain other cosmological parameters, such as the equation of state parameter w (for example, [8]).

4.3 Ineffectiveness of Flux Limited Cluster Counts for Dimming Mechanisms

As we mentioned earlier, we cannot use cluster counts to constrain dimming mechanisms. This is not to say that dimming mechanisms do not affect cluster counts; indeed, dimming would lead to fewer clusters above the flux limit in any given survey. However, the effect of dimming would only be through a modification of the observed evolution of cluster luminosities, which is currently not well constrained theoretically. To gauge the effect that photon-axion oscillations could have on cluster counts we can assume some particular intrinsic $L - T$ evolution, and examine the predicted number counts with and without oscillations. Figure 3 shows number counts versus redshift for different values of $L_{\rm dec}$, where $L_{\rm dec} = \infty$ corresponds to no photon-axion oscillations and values of $L_{\rm dec}$ as low as 30 Mpc correspond to roughly 1/3 of the light lost. The curves correspond to $\Omega_m = 0.3$ and $\sigma_8 = 0.9$ with $\Gamma = 0.1$ and $\mathrm{T}_{15} = 6$ keV. We assumed here that the intrinsic luminosity evolution is specified by the parameters (2.22), although there is no theoretical justification for this.

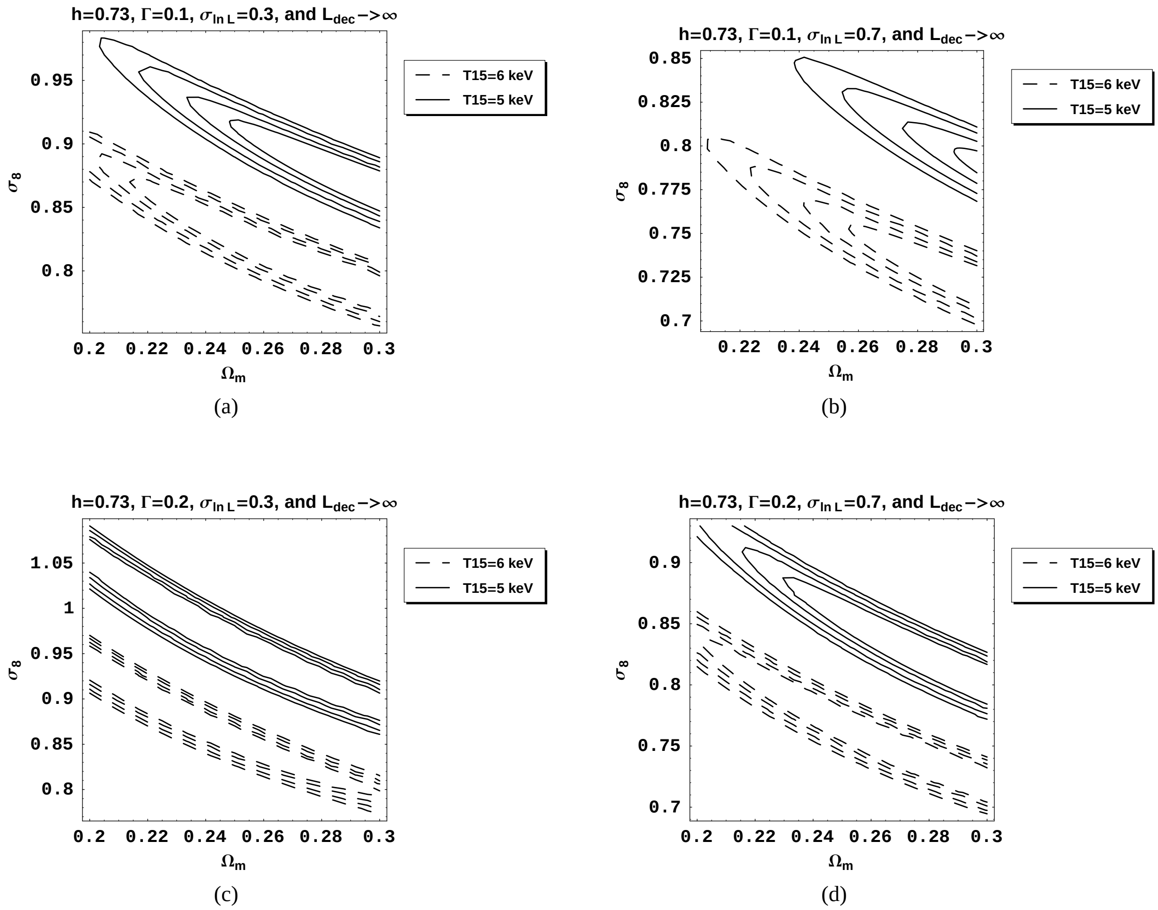


Figure 2: Confidence plot of Ω_m and σ_8 for various choices of model parameters. The lines represent 68%, 80%, 90%, and 95% confidence regions. (a) $\Gamma = 0.1$, $\sigma_{\ln L} = 0.3$. (b) $\Gamma = 0.1$, $\sigma_{\ln L} = 0.7$. (c) $\Gamma = 0.2$, $\sigma_{\ln L} = 0.3$. (d) $\Gamma = 0.2$, $\sigma_{\ln L} = 0.7$

The fact that the observed luminosity evolution is used as input in this analysis implies that the constraints on cosmological parameters from Section 4.2 are valid, independent of any dimming mechanism. As a consequence, cluster constraints on the equation of state parameter w can be compared with constraints from Type Ia supernova surveys, which *would be* affected by dimming mechanisms, as in Refs. [22]. Such a comparison would then provide a new test of the photon-axion oscillation model. It would also be interesting to compare with other constraints on photon-axion oscillations, for example from CMB spectral distortion [56].

4.4 Flux Limited Cluster Counts for Other Types of New Physics

Although we do not attempt further analyses here, flux limited cluster counts are more suitable for constraining new physics that modifies structure formation as opposed to the apparent luminosity of clusters. There are many well-known examples of such possible new physics. These include possible new interactions in the dark sector and new light species

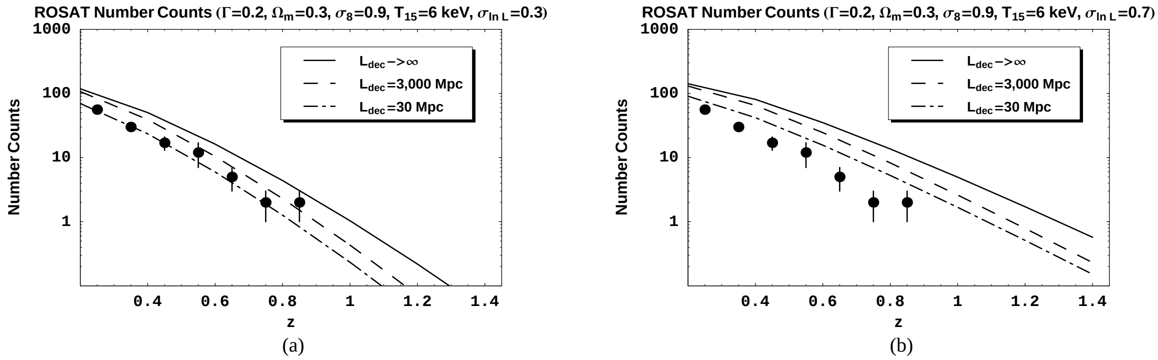


Figure 3: Number Counts versus redshift for $\Omega_m = 0.3$ and $\sigma_8 = 0.9$ including axion oscillations for a fixed intrinsic L-T relation, with different levels of scatter in the L-M relation. (a) $\sigma_{\ln L} = 0.3$. (b) $\sigma_{\ln L} = 0.7$.

that would wash out structure. Cluster surveys would also be useful in constraining such phenomena as late time phase transitions in the dark sector and other aspects of possible new gravitational dynamics. It would be straightforward to build these types of new physics into the formalism described above.

5. Significance of Future Cluster Surveys

It is important to recognize that studies of cluster evolution, while already interesting, will continue to develop. In this section, we briefly mention some future approaches that will enhance our knowledge of cluster growth, and note the impact of dimming. In particular, Sunyaev-Zeldovich (SZ) surveys such as the South Pole Telescope (SPT) [57] or the Atacama Cosmology Telescope (ACT) [58] will establish catalogues of clusters which are unbiased in redshift - a crucial element difficult to achieve with X-ray surveys. The Dark Energy Survey (DES) [59] will take advantage of the SPT survey, and include photometric redshifts, as well as lensing measurements of cluster masses, and other, independent tests of cosmology. The Large Synoptic Survey Telescope (LSST) [60] will provide masses of a huge set of clusters via weak lensing tomography. Future x-ray surveys can expand tremendously the statistics and knowledge of many of the uncertainties described in earlier sections, in particular the evolution of cluster properties [61].

One of the key difficulties in using X-ray surveys to extract cosmology, especially within the present context of photon-axion oscillations, is the indirect, and uncertain relationship of luminosity to temperature and temperature to mass. Future studies will mitigate these issues.

SZ surveys will be a tremendous source of new information in the near future. For a review, see [62]. The SZ effect is a decrement ΔT in the CMBR given by the line of sight integral

$$\frac{\Delta T}{T_{CMBR}} = -2\frac{\sigma_T}{m_e c^2}\int dl\, n_e(l) k_B T_e(l) \tag{5.1}$$

where σ_T is the Thomson cross section, m_e is the electron mass, n_e is the electron number density and T_e is the electron temperature. Clusters, with masses in excess of $10^{14} M_\odot$, have sufficient gas densities and temperatures that large scale surveys are possible. The key feature of the SZ effect is the perturbation of the CMBR which is independent of redshift, and thus allows for a cluster sample, without concern of the selection issues associated with luminosity-weighted X-ray surveys described in earlier sections.

The SZ effect is not proportional to mass alone, but to the electron pressure. Extracting the mass is a challenge, and of vital importance if these surveys are to provide precision limits on cosmology [63, 64, 65]. Techniques can involve self calibration [66, 67, 64, 68], measurements of the cosmic shear (as in the DES or LSST), or complementary measurements of the cluster x-ray temperature.

As already noted, microwave studies, such as SZ surveys, should not be impacted by dimming mechanisms. Hence, the appearance and properties of clusters within such experiments should be a robust test of the dark energy properties. Similarly, surveys employing weak lensing will also determine mass and redshift properties will also be insensitive. Supernovae, on the other hand, acting as standard candles, are clearly impacted. As a consequence, studies of cluster growth are key tools of cosmology and measure a quantity *distinct* from that of supernovae, quite in the way envisioned by Wang, et al [21].

6. Conclusions

We have reviewed some of the current models of structure formation and galaxy cluster dynamics relevant for comparing cluster surveys with models of particle physics and gravity. We compared predictions in the standard ΛCDM cosmology to the 400 Square Degree ROSAT galaxy cluster survey, and found that, with a relatively large assumed scatter in the relation between cluster mass and temperature, our analysis is consistent with the WMAP 3-year data. Earlier analysis of the HIFLUGCS cluster survey indicates even better agreement with CMB data [20].

We studied a model of cluster dimming by photon-axion oscillations, and found that a better theoretical understanding of cluster luminosity evolution is required before firm conclusions could be drawn regarding dimming mechanisms using cluster data. In particular, improvements in theoretical models and experimental measurements of the evolution of the luminosity-temperature relation may provide an important test of such mechanisms in the future. Moreover, we noted that the cosmological parameters extracted from cluster count surveys are independent of dimming mechanisms, given the measured $L - T$ relation, in contrast to supernovae, and thus provide an independent test of such models. This suggests that such surveys should be folded into analyses such as [21] in order to additionally constrain them. Although cluster counts are insensitive to dimming mechanisms, it is possible to impose relatively strong constraints on models of new physics that would affect structure formation as opposed to cluster appearances. This includes any physics that would alter the overall expansion rate of the universe, the existence of light species that would help to

wash out structure, and additional interactions in the dark sector that could either encourage or inhibit the growth of structure. The effects of new physics on structure formation can straightforwardly be built into the Press-Schechter formalism.

The simple models that enter our fits rely on comparison to numerical simulations for justification, and deserve to be scrutinized. For example, a higher than typical normalization T_{15} in the $M-T$ relation, while not justified by simulations, could bring our fits still better in line with the WMAP 3-year data. Also, the level of scatter in the $L-T$ and $M-T$ relations deserves to be better analyzed, since a high assumed scatter brings our fits better in line with the WMAP fits. Despite the remaining uncertainties in this formalism, galaxy cluster surveys are increasingly ripe for their utilization in constraining new physics. Upcoming Sunyaev-Zel'dovich and weak-lensing cluster surveys hold the promise of a better determination of the cluster mass function, and should help to eliminate some of the current uncertainties in these techniques.

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